

METHODOLOGY OF TEACHING BASIC KNOWLEDGE IN PROBABILITY THEORY AND MATHEMATICAL STATISTICS

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Abstract. The article addresses the teaching of fundamental concepts in probability theory and mathematical statistics. Specifically, it discusses such concepts as experiment, realization of an experiment, outcome of an experiment as an elementary event, event, and probability space. An original auxiliary model called 'Market', proposed by N. Temirgaliyev, is introduced for studying the probability space of an experiment. Using the example of the standard experiment 'coin tossing' and the culturally rooted 'asyk tossing' (a traditional Kazakh game), auxiliary and probability spaces are constructed.

Additionally, the article presents a general scheme for constructing a probability space when the set of elementary events is finite. The practical significance of probability is also explored. A sequence of random numbers is generated, and based on it, a model of an experiment involving the tossing of a symmetrical and homogeneous die is developed. The frequency of a specific face of the die appearing is calculated for both a random sample and a series of experiments. In both cases, the Law of Large Numbers is confirmed, demonstrating the convergence of frequencies toward the event's probability as the number of trials increases.

Furthermore, the article discusses the methodology for presenting a general framework for constructing a probability space when the set of elementary events is finite, as well as methods for demonstrating the practical relevance of probability. A sequence of random numbers is again generated to model an experiment involving the tossing of a symmetrical and homogeneous die, and the frequency of a fixed face appearing is described for a random sample and for experimental series. In both instances, learners are given the opportunity to take their first scientific steps by observing the Law of Large Numbers in action, showing how the frequency of an event approaches its theoretical probability as the number of experiments increases.

Keywords: teaching methodology, probability theory and mathematical statistics, probability model, auxiliary “Market” model, practical significance of probability.

Introduction

David Mumford (Mumford, 2005) highlighted the future importance of probability theory and mathematical statistics with the following observation: “In the future, probability and statistics will be seen as natural tools for working in both mathematical and scientific modeling. ... Probability theory and the statistical approach have gradually come to be viewed as the best foundations for scientific models...”. Today, probability theory and mathematical statistics are evolving not only as independent scientific disciplines but also as the foundational framework for a wide range of emerging applied fields. These include machine learning, artificial intelligence, data science, biostatistics, psychometrics, cyberstatistics, and others. Consequently, core concepts from probability and statistics have been integrated into school curricula in many countries around the world, reflecting their growing significance in both education and modern scientific practice.

In Kazakhstan’s secondary schools statistics and probability theory are included in the curriculum of mathematics and algebra in grades 5-11 (Secondary school textbooks, 29 2005), while in Russia, starting in 2022, in grades 7-11, mathematics, along with algebra and analysis basics and geometry courses, will be supplemented with a third course called probability and statistics (On amendments to the Federal State Educational Standard, 2025). As for Singaporean educational programs (Mathematics syllabus, 2025a), they include the elements of statistics included in the domestic educational programs, and in the upper grades, they are also supposed to teach the topics of hypothesis testing, correlation and linear regression (Mathematics syllabuses, 2025b), (Mathematics syllabus Pre-University 36 Higher, 2025). The scope of statistics in general schools of the People’s Republic of China is comparable to the content in domestic programs. In advanced mathematical schools, interval estimation, hypothesis testing, elements of regression analysis, and analysis of variance (Anova) are also taught (Khan Academy, 2025).

The Texas Education Agency has developed a textbook (Illowsky & Dean, 2020) to provide regions with high-quality textbooks free of charge, covering topics such as sampling and databases, descriptive statistics, probability theory, discrete and continuous random variables, normal distribution, central limit theorem, confidence intervals, single and double hypothesis testing, chi-square distribution, linear regression and correlation, F distribution, and analysis of variance (ANOVA). The textbook includes a brief theoretical overview of the topics, a list of real-life examples and problems for each topic.

The subject of probability theory by A.N. Shiryayev (Shiryayev, 1980) is the mathematical analysis of random phenomena, i.e., such phenomena are studied where there is no deterministic regularity (observations of them do not always lead to the same outcomes) and at the same time, they have some statistical regularity. Probability theory itself, like any exact science, became such only when the concept of a probabilistic model was clearly formulated in the axiomatization of science by A.N. Kolmogorov (Kolmogoroff, 1933).

Probability Theory studies various random phenomena with a set of all possible outcomes of an experiment in the form of finite, countable and more than countable measurable sets. Series Theory is sufficient to study models of experiments with a countable set of outcomes. The highest generality of Probability Theory is the uncountable power of the set of all possible outcomes of an experiment, which should be preceded by the Theory of Measure and the Lebesgue-Stieltjes Integral. The article studies the methodology of teaching some basic concepts of probability theory and mathematical statistics, limited to experiments with a finite number of outcomes.

Methods

A review was conducted on the methodology of teaching some basic concepts of Probability Theory and Mathematical Statistics within secondary school curricula, using theoretical methods such as analysis and synthesis, as well as content analysis, which made it possible to highlight key features. In addition, a method of modeling real-life experiments was applied to support the understanding of fundamental concepts. Furthermore, the action research method was employed in the classroom to reinforce the material by applying the methodologies presented in the article.

Results and discussion

Methods of teaching basic knowledge in probability theory and mathematical statistics. To study various phenomena using mathematical apparatus, mathematical models are built. In his work (Kolmogoroff, 1933), published in German in Berlin in 1933, Andrei Nikolaevich Kolmogorov axiomized probability theory into a branch of mathematics, based on the theory of integration introduced in 1902 by the French mathematician Henri Lebesgue in his famous dissertation "Integral, Length, and Area" (Lebesgue, 1902). The system of axioms proposed in this work describes the experiment by means of a mathematical model – the Probability space $(\Omega, \sigma A, P)$ (and (Ω, A, P) in the case of finite Ω) consisting of three parts:

- space of elementary events Ω – a set consisting of all possible outcomes of the experiment, called elementary events ω ,
- $A(\sigma A)$ is an algebra (sigma-algebra) of measurable subsets, called events, of the set Ω ,
- probability P is a non-negative additive (sigma-additive) function of sets, defined on the $A(\sigma A)$ algebra (sigma-algebra) with the condition $P(\Omega)=1$.

Within the framework of probabilistic models with a finite number of outcomes, it is possible to construct the entire Probability Theory in all its main details, with its further extension to countable and continuous cases. Probabilistic models with a finite number of outcomes are sufficient in studying this subject in secondary school and in specialties whose educational programs do not include Measure Theory and the Lebesgue integral.

The computational technical apparatus in Probability Theory is the introduction of a probability measure on a pair consisting of the space of elementary events Ω and the algebra (sigma-algebra) of events $A(\sigma A)$. In the very initial case, in Elementary Probability Theory, the measure on Ω consists of finite sums of real numbers, when each element of this finite set is assigned a non-negative number, called its probability, which in sum give one. This requirement is called normalization and is introduced only for the sake of uniformity throughout Probability Theory. The technique here is the simplest – an event consists of a finite number of elementary events with their own probability, the probability of an event is a finite sum of the probabilities of the elementary events that make it up.

In the discrete case, when the space of elementary events Ω is finite or countable, the measure is defined on all subsets of the set Ω . Thus, each subset of the space of elementary events Ω forms an event whose probability measure is equal to the sum of the probability measures of all the elementary events that make it up. When Ω is a more than countable set, the probability is defined on the algebra (sigma-algebra) of measurable subsets $A(\sigma A)$ of the set Ω the study of which requires a deeper Theory of Measure and the Lebesgue-Stieltjes Integral. Further, on the basis of the probability triple (Ω, A, P) , all fundamental concepts of probability theory are defined with an extension to Mathematical Statistics.

The presentation of Probability Theory in the case of experiments with a finite number of outcomes is motivated in nature for each of the fundamental concepts of Probability Theory with the most extensive explanations possible, sufficient for understanding the main content of the subject.

Methodology of teaching the concepts «Experiment, implementation of the experiment and its outcomes». From the authors' 20 years of experience in teaching this subject, it can be stated that most students do not distinguish between basic concepts such as experiment, implementation of an experiment, all possible outcomes of an experiment, make mistakes in constructing the space of elementary events Ω , in describing the set of all possible events and determining probability. Therefore, in each study of experiments, it is necessary to describe its mathematical model (Ω, A, P) .

Let us present mathematical definitions and methodological recommendations for teaching these basic concepts.

The first stage consists of distinguishing between such concepts as an experiment, the implementation of an experiment, and a variety of possible outcomes of an experiment. To do this, in the initial classes it is necessary to fill in Table 1 in writing and after fundamental assimilation it is sufficient to pronounce it orally.

Table 1. General scheme to distinguish three initial concepts

Experiment	
Implementation of the experiment	
The set of all possible outcomes of the experiment	

Tossing coins and throwing dice are model examples that can also be easily implemented in lessons by the students themselves. As a first example, let's consider tossing coins and fill in Table 1 for it.

Table 2. Distinguishing concepts in the experiment of tossing a coin once

Experiment	Tossing a coin
Implementation of the experiment	The coin is tossed once
The set of all possible outcomes of the experiment	<i>(the side of the coin marked with a head),</i> <i>(the other side)</i>

Along with standard examples, the use of national elements (e.g., national games) illustratively facilitates the assimilation of the topic. For example, consider the famous Kazakh national game "Asyk atu", which is translated as something like "to beat out with one's own asyk". "Asyk" is a bone of a sheep's knee and "atu" translates as beat out. Asyk atu has four outcomes when tossed, which is called – "buge", "shige", "alshi", "tayki" (see Fig. 1). Here, each outcome has its own probability of falling out and other possible outcomes are not considered. Table 1 for tossing "asyk" is presented in Table 3.

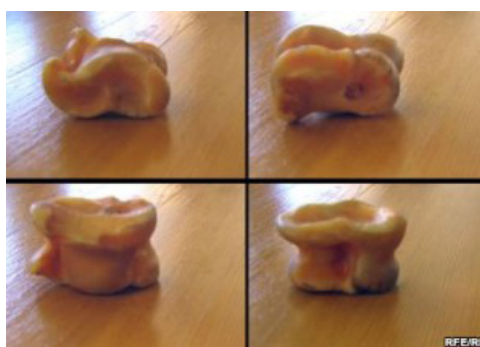


Figure 1. All possible positions of the outcomes when throwing "asyk".

Table 3. “Asyk” concepts in the practice of tossing a stick once

Experiment	Asyk atu
Implementation of the experiment	The «asyk» is tossed once
The set of all possible outcomes of the experiment	$\omega_1=alshi, \omega_2=tayki,$ $\omega_3=buge, \omega_4=shige$

During the teaching process, giving tasks that increase the level of knowledge of the customs and traditions of the people, as in the example above, along with raising national consciousness, covers the educational aspect of the subject in addition to the knowledge. However, such connections with other fields and customs should not overshadow the content of the topic, but should be additional information.

It is also a good idea to provide various practical examples so that students do not think that the experiments considered in probability theory consist only of tosses. For example, consider an example in which one boy and one girl are selected as the organizers of an event from three boys and two girls in the class. Let's list all the possible outcomes here.

Experiment: One boy and one girl are selected from a class of five children (three boys – e_1, e_2, e_3 ; two girls – g_1, g_2).

All possible outcomes of the experiment: $\omega_1=(e_1, g_1), \omega_2=(e_1, g_2), \omega_3=(e_2, g_1), \omega_4=(e_2, g_2), \omega_5=(e_3, g_1), \omega_6=(e_3, g_2)$. The order of the two children selected in these outcomes is not important.

In the examples given above, the number of all possible outcomes of the experiments was finite. It was also mentioned above that if the curriculum includes series and measure theory, cases where the number of possible outcomes of the experiment is more than finite are also considered.

And so, in all cases of the considered experiments, Table 1 should be filled in writing until the students clearly distinguish between the experiment, the implementation of the experiment and all possible outcomes of the experiment. Next, a mathematical model of the experiment is constructed – a probabilistic triple Ω, A, P . When studying this topic in high school or even in a university program, together with the mathematical model, it is recommended to use the auxiliary model “Market”, proposed in (Mathematics syllabus, 2025a), as the author's method of teaching probability theory.

About the author's approach to teaching the basic concepts of probability theory – the auxiliary model “Market” and the implementation of this idea. Let us imagine a retail outlet – “Market”, which has different goods, each in a single copy and with its own price for a total of 1 unit (any amount can be taken as a unit, for example, 407591 tenge and any others, it is enough to assign a new price to each product equal to the ratio of the old price to the price of all goods in the “Market”). The prices of the goods can coincide, unlike the goods themselves, while the uniqueness of each product is required in the mathematical model when using the concept of “Set of elementary events”, since any object “belongs” or “does not belong” to this set.

Let's make up all possible packages from any goods of the Market – from an empty package to the contents of the entire Market. Then the price of the package “The whole Market” will be equal to the sum of the prices of all goods of the package, and, as agreed here, equal to 1.

This is what we will call the “Auxiliary Model “Market””, namely, for a given fixed experiment, each of its outcomes (result, observation) corresponds to exactly one product in the Auxiliary Model “Market” with all the ensuing consequences – the price of the product, the package of products, the price of the package.

When studying mathematics, it is always useful to remember that it is a systematized science and therefore is expressed in precise definitions. So, if in the example with Market we

replace the usual names with special mathematical terms, we will get the necessary theoretical basis for understanding Probability Theory - a probabilistic model:

- 1) A product is an elementary event ω ;
- 2) The price of a product is the probability $P(\{\omega\})$ of an elementary event ;
- 3) A package made up of some goods – event A , made up of elementary events corresponding to the goods of the package;
- 4) The price of a package is the probability of an event $P(A)$ numerically equal to the sum of the probabilities of all elementary events from which the event is composed;
- 5) Market – a set composed of all elementary events.

Briefly in notation:

- sample space – space of elementary events $= \{\omega_1, \dots, \omega_n\}$ of elements,
- elementary event,
- event A composed of elementary events $\omega_{i_1}, \dots, \omega_{i_k}$ with numbers $1 \leq i_1 < \dots < i_k \leq n$, $A = \{\omega_{i_1}, \dots, \omega_{i_k}\}$, or an empty set,
- probability P is a regular numerical function on the set of events $P(\{\omega\})$ with values on elementary events the probability of occurrence of an elementary event such that $0 \leq P(\{\omega\}) \leq 1$, probability $P(A)$ of event A as the price of a packet is equal to the sum $P(\{\omega_{i_1}\}) + \dots + P(\{\omega_{i_k}\})$, $(\Omega) = P(\{\omega_1\}) + \dots + P(\{\omega_n\}) = 1$ as the price of the entire Market.

The following table shows the analogy between the probabilistic and auxiliary models.

Table 4. Probability model and auxiliary model “Market”

Probability model	Auxiliary model «Market»
<i>Elementary events</i>	<i>Goods</i>
$\omega_1, \omega_2, \dots, \omega_n$	
Probability of an elementary event	Price of the goods
$P(\{\omega_1\}) = p_1, P(\{\omega_2\}) = p_2, \dots, P(\{\omega_n\}) = p_n$	
Event A , consisting of elementary events	Package A of some goods
$A = \{\omega_{i_1}, \dots, \omega_{i_k}\} (k=1, 2, \dots, n)$ or empty package $A = \emptyset$	
The probability of event – A – $P(A)$	$P(A)$ – The price of package – A – $P(A)$
$P(A) = P(\{\omega_{i_1}, \dots, \omega_{i_k}\}) = P(\{\omega_{i_1}\}) + \dots + P(\{\omega_{i_k}\}) = p_{i_1} + \dots + p_{i_k},$ $P(\emptyset) = 0,$ $p_1 + p_2 + \dots + p_n = 1.$	
Sample space – the set which consist all elementary events	Market
$\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$	

In the course of studying probability theory, the next concept to pay attention to in the triple (Ω, A, P) is an event, that is, any element of the event algebra A . It is important to present the definition of an event using mathematical tools so that it can be understood and further studied. Let us give the definition of an event for any set of elementary events Ω .

Definition. Any measurable subset of the set of all possible elementary events is called an event, that is, an event is a measurable set consisting of elementary events: $A \subset \Omega, A$ is an event.

And if the set of elementary events Ω is a finite set, the definition of an event can be given as follows without using the concept of measurability.

Definition. Any subset of the set of all possible elementary events is called an event, that is, an event is the set consisting of elementary events: $A \subset \Omega, A$ is an event.

It was mentioned above that the structure of probability theory in school mathematics can be fully described without the concept of measure if the number of elementary events is finite. Understanding the definition given from this perspective does not cause significant difficulties for pupils who are familiar with the concept of a set from the 2nd grade, and it is possible and easy to verify the definition of subsets of a set mathematically. According to the presented auxiliary "Market" model, they visually understand it in terms of a package, and this concept is fixed and retained in the minds of students.



Auxiliary and probabilistic model of experiments on the example of one-time coin toss and one-time asyk toss. So, we have two models: the auxiliary model "Market" and the same in probabilistic terms, the probability model.

Let's make these two models for experiments: one-time coin toss with sides H (Head), T (Tail) and asyk tossing with four possible outcomes.

Auxiliary model "Market" of the experiment "one-time coin toss". All possible results of the toss form goods H and T and market $\{H; T\}$.

Prices of goods (set arbitrarily): the price of good H is equal to a non-negative number p , the price of good T is equal to a non-negative number q . The price of all goods, the entire market, is equal to $p+q=1$.

Packages: empty $\emptyset$; consists of one H ; of one T ; of H and T – the whole market. And, as a consequence, the prices of packages: empty equals 0; of one H equals p ; of one T equals q ; of H and T equals 1 (see Table 5).

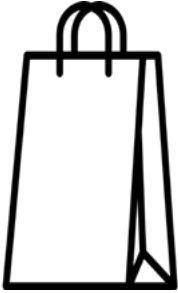
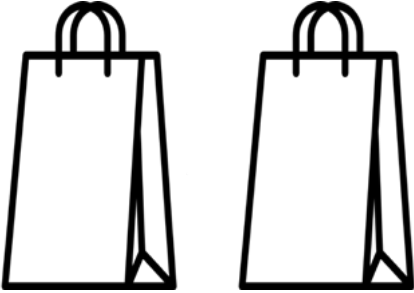
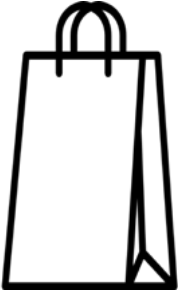
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Packages: empty $\emptyset$; consists of one H ; of one T ; of H and T – the whole market. And, as a consequence, the prices of packages: empty equals 0; of one H equals p ; of one T equals q ; of H and T equals 1 (see Table 5).

Table 5. All possible packages (events) of experiments of one-time coin toss

		
Empty package	A packet consisting of only one H A packet consisting of only one T	A package consisting of all possible goods – H and T

The same in probabilistic terms:

Probabilistic model of the experiment "one-time coin toss":

All possible elementary events $\omega_1 = H$ and $\omega_2 = T$ form the space of elementary events $\Omega = \{\omega_1; \omega_2\}$.

All possible events: $A_0 = \emptyset, A_1 = \{\omega_1\}, A_2 = \{\omega_2\}, A_3 = \{\omega_1; \omega_2\} = \Omega$.

Probabilities: probabilities of elementary events $P(\{\omega_1\}) = P(\{H\}) = p$ and $P(\{\omega_2\}) = P(\{T\}) = q$, and, as a consequence, probabilities of events $P(A_0) = 0, P(A_1) = p, P(A_2) = q, P(A_3) = p + q = 1$.

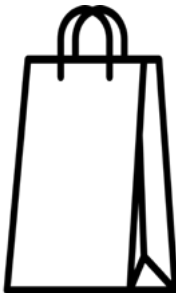
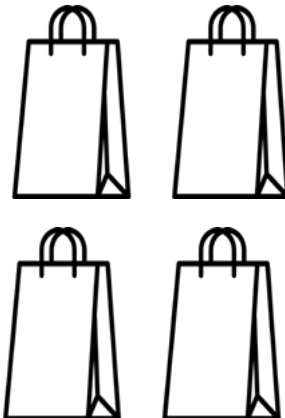
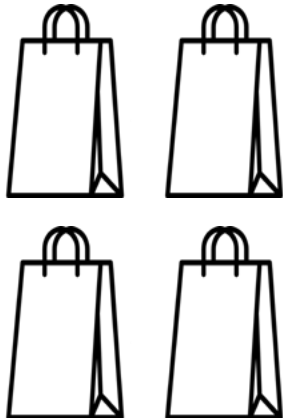
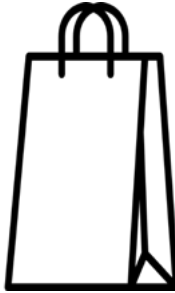
Now we will present the auxiliary and probabilistic models of the experiment for tossing asyk.

Auxiliary model "Market" of the experiment "Tossing asyk". The entire market consists of four goods: alshi - a, teyki - t, buge - b, shige - sh with prices, non-negative numbers - p_1, p_2, p_3, p_4 respectively, where $p_1 + p_2 + p_3 + p_4 = 1$ units.

We list the packages and their prices:

- empty $\emptyset - 0$,
- with one product: {buge} – p_1 , {shige} – p_2 , {alshi} – p_3 , {teyki} – p_4 ,
- with two goods: {buge, shige} – $p_1 + p_2$, {busge, alshy} – $p_1 + p_3$, {shige, alshy} – $p_2 + p_3$, {buge, teyki} – $p_1 + p_4$, {shige, teyki} – $p_2 + p_4$, {alshy, teyki} – $p_3 + p_4$;
- with three goods: {buge, shige, alshy} – $p_1 + p_2 + p_3$, {buge, shige, teyki} – $p_1 + p_2 + p_4$, {buge, alshy, teyki} – $p_1 + p_3 + p_4$, {shige, alshy, teyki} – $p_2 + p_3 + p_4$,
- and, finally, with four products - the entire market {buge, shige, alshy, teyki} – $p_1 + p_2 + p_3 + p_4 = 1$ (see Table 6).

Table 6. All possible packages (events) of experiments of one-time asyk toss

		
Empty package	Package of one a Package of one t Package of one b Package of one sh	Package of a and t , Package of a and b Package of a and sh , Package of t and b Package of t and sh , Package of b and sh
		
Package of a , t , b Package of a , t , sh Package of a , b , sh Package of t , b , sh		Package of a , t , b , sh

Probabilistic model of the experiment “one-time asyk toss”:

Sample space Ω is a set composed of all elementary events. All possible elementary events $\omega_1 = \text{buge}$, $\omega_2 = \text{shige}$, $\omega_3 = \text{alshi}$, $\omega_4 = \text{teyki}$ form the space of elementary events $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. Probabilities of elementary events: $P(\{\omega_1\}) = p_1, P(\{\omega_2\}) = p_2, P(\{\omega_3\}) = p_3, P(\{\omega_4\}) = p_4$.

2) All possible events (subsets of the space of elementary events):

$$A_1 = \emptyset, A_2 = \{\omega_1\}, A_3 = \{\omega_2\}, A_4 = \{\omega_3\}, A_5 = \{\omega_4\}, A_6 = \{\omega_1; \omega_2\}, A_7 = \{\omega_1; \omega_3\}, A_8 = \{\omega_1; \omega_4\}, A_9 = \{\omega_2; \omega_3\}, A_{10} = \{\omega_2; \omega_4\}, A_{11} = \{\omega_3; \omega_4\}, A_{12} = \{\omega_1; \omega_2; \omega_3\}, A_{13} = \{\omega_1; \omega_3; \omega_4\}, A_{14} = \{\omega_1; \omega_2; \omega_4\}, A_{15} = \{\omega_2; \omega_3; \omega_4\}, A_{16} = \{\omega_1; \omega_2; \omega_3; \omega_4\} = \Omega$$

$$\text{or} \quad A_1 = \emptyset, A_2 = \{\omega_1 = \text{alshi}\}, A_3 = \{\omega_2 = \text{teyki}\}, A_4 = \{\omega_3 = \text{buge}\},$$

$$A_5 = \{\omega_4 = \text{shige}\}, A_6 = \{\omega_1 = \text{alshi}, \omega_2 = \text{teyki}\}, A_7 = \{\omega_1 = \text{alshi}, \omega_3 = \text{buge}\},$$

$$A_8 = \{\omega_1 = \text{alshi}, \omega_4 = \text{shige}\}, A_9 = \{\omega_2 = \text{teyki}, \omega_3 = \text{buge}\},$$

$$A_{10} = \{\omega_2 = \text{teyki}, \omega_4 = \text{shige}\}, A_{11} = \{\omega_3 = \text{buge}, \omega_4 = \text{shige}\},$$

$$A_{12} = \{\omega_1 = \text{alshi}, \omega_2 = \text{teyki}, \omega_3 = \text{buge}\},$$

$$A_{13} = \{\omega_1 = \text{alshi}, \omega_2 = \text{teyki}, \omega_4 = \text{shige}\},$$

$$A_{14} = \{\omega_1 = \text{alshi}, \omega_3 = \text{buge}, \omega_4 = \text{shige}\},$$

$$A_{15} = \{\omega_2 = \text{teyki}, \omega_3 = \text{buge}, \omega_4 = \text{shige}\},$$

$$A_{16} = \Omega = \{\omega_1 = \text{alshi}, \omega_2 = \text{teyki}, \omega_3 = \text{buge}, \omega_4 = \text{shige}\}$$

3) Probabilities of events (numerical functions whose arguments are events):

$$\begin{aligned} P(A_1) &= 0, P(A_2) = p_1, P(A_3) = p_2, P(A_4) = p_3, P(A_5) = p_4, P(A_6) \\ &= p_1 + p_2, P(A_7) = p_1 + p_3, P(A_8) = p_1 + p_4, P(A_9) \\ &= p_2 + p_3, P(A_{10}) = p_2 + p_4, P(A_{11}) = p_3 + p_4, P(A_{12}) \\ &= p_1 + p_2 + p_3, P(A_{13}) = p_1 + p_2 + p_4, P(A_{14}) \\ &= p_1 + p_2 + p_4, P(A_{15}) = p_2 + p_3 + p_4, P(A_{16}) = P(\Omega) \\ &= p_1 + p_2 + p_3 + p_4. \end{aligned}$$

By creating such models, students have the opportunity to understand each concept in the probability space. Most students have questions such as how an event can be an empty set, how it somehow lands on one side when tossing a coin, or how an event can contain all possible outcomes and only land on one side, which indicates that the student does not understand the definition, has not reached the level of “mathematical maturity”. Only when an event is perceived as a subset of all possible elementary events, for example, in the experiment of tossing a coin once, does he understand that by definition each of the sets $A_1=\emptyset, A_2=\{\omega_1\}, A_3=\{\omega_2\}, A_4=\{\omega_1;\omega_2\}=\Omega$ is an event, which is visually captured in the mind through the package.

It is known from set theory that the number of elements in a set is equal to the power of 2 in the system of sets consisting of all its possible subsets. It is a very tedious and unnecessary task to write down all its possible events for each experiment. Therefore, after distinguishing each concept from the others and understanding its meaning, it is enough to write down the set of all possible elementary events and the event under study.

General scheme of building a finite probability model - "Probability space". Summarize the results. With a proper understanding of the detailed and, if necessary, additional examples (for example, to fully describe the auxiliary and probabilistic model of a double toss of a coin, tossing a dice, etc.), it is advisable to skip the "Auxiliary Model" and go directly to the "Probability Model".

In the light of the given information, I will once again clarify the position in the Theory of Probabilities. We consider the task to be an experiment, which consists in the fact that a test is carried out, and in any limited quantity, the outcome of each of which is not known in advance, but the whole set of possible outcomes is known, the occurrence or non-occurrence of which must be observed. At the same time, the set of conditions under which the experiment is carried out is stipulated in advance, and these conditions are either created artificially or carried out independently of the will of the experimenter. The probability space $(\Omega, \mathcal{F}, P)$ is a mathematical modeling experiment

This is important for many reasons. One of them is the selection of basic information from which all the rest can be obtained - a kind of reduction to the possible amount of necessary information.

In our simplest initial scheme, as well as in the extensive Probability Theory, all possible test results are known in principle, but it is not known which of them is realized in this test.

A probabilistic problem, with a given experiment, arises when the result of the test, obeying the will of the case, is not known in advance, - by chance.

The general scheme of building a finite probability model for a given experiment with a finite number of outcomes:

1) Write down all the possible results, elementary events of the implementation of the experiment: $\{\omega_1, \dots, \omega_n\}$ - a set of elementary events. Elementary events are also called cases in mathematical literature.

2) Each elementary event ω_i ($i=1, \dots, n$) is assigned a non-negative number p_i - its probability: $P(\{\omega_i\})=p_i$, but with the condition $p_1+p_2+\dots+p_n=1$.

3) Event A is called an arbitrary subset of sets, i.e. a set composed of elementary events - elements of : from 0 elements, from 1 element, from 2 elements, ..., from n elements. Thus, the event consists of elementary events.

An elementary event belonging to event A is also called a case favorable to event A .

An elementary event that does not belong to event A is also called a case that does not favor event A .

4) Probability $P(A)$ of event A is the sum of elementary events that make up its probability: if $A=\{\omega_{i_1}, \dots, \omega_{i_k}\} (1 \leq k \leq n; 1 \leq i_1 < i_2 < \dots < i_k \leq n)$, then $P(A)=p_{i_1} + \dots + p_{i_k}$.

And, as a result, the usual property of additivity, when the whole is assembled from its components

5) If events A and B do not have common elementary events, then the probability of an event composed of non-intersecting sets A and B is equal to the sum of the probabilities of A and B .

Three components are distinguished in the presented probabilistic model: the space of elementary events - sample space Ω ; set $\mathcal{A}$ of all possible events A are subsets of sets Ω ; probability $P: \mathcal{A} \rightarrow [0,1]$ is a numerical non-negative additive function of sets on $\mathcal{A}$, normalized by the condition $P(\Omega)=1$.

This triple $(\Omega, \mathcal{A}, P)$ is called a probability space.

Attracting students to science with the example of numerical confirmation of the statistical definition of probability. On the statistical regularity of experiments.

After the probability triad is given, the practical meaning of probability is discussed to answer the questions of what probability is and what it means. More precisely, an event is given, its probability is determined. What does this knowledge give us? Let's answer this question.

Repeating the experiment with the students a certain number of times and monitoring how many times a given event has occurred. Then the frequency as the ratio of the number of occurrences of an event to the number of experiments conducted, is close to the probability of a given event, and the closeness increases with the number of experiments, which is the content of the law of large numbers of Jacob Bernoulli, created in 1713. Of course, the pupils are not familiar with this, but a school teacher who has this knowledge explains it to the pupils at an accessible level.

The practical significance of probability is given by experiments with tossing a symmetrical and homogeneous coin.

Any real coin is not absolutely symmetrical and uniform, and the production of a coin close to ideal requires correspondingly greater efforts. Therefore, experiments with coins are considered as a scientific result, which is also one of the topics for attracting pupils to science.

Thus, the probabilities of getting a coat of arms and a hash mark, due to the proximity of the coin to ideal, are taken to be equal, i.e. $p = \frac{1}{2}$ for each of the sides. This is confirmed by the results of experiments in various historical periods by researchers.

Table 5 presents the well-known results of experiments involving a single coin toss. The coin is tossed sequentially. Event is recorded, in this case, "heads coming up", and for each realization the result "event occurred" and "event did not occur" are noted. Next, the ratio of the number of results "event occurred" to the number of all realizations is taken - in this case, the proportion of heads coming up" and compared with the expected number of 0.5 - the probability of event .

Table 7. Results of experiments on single coin tossing

Experimenter	Number of experiments (throws)	Frequency of head appearance
Buffon (1707-1788) – French naturalist, biologist, mathematician, natural scientist	4040	0,507
De Morgan (1806-1871) – Scottish mathematician, logician	4090	0,505
Feller W. (1906-1970) – American mathematician	10000	0,4979
Jevons (1835-1882) – English economist, philosopher	20480	0,5068
Pearson K. (1857-1936) – English mathematician, statistician	24000	0,5005
Romanovsky V.I. (1879-1954) – Soviet mathematician	80640	0,4933

Apparently, all these experiments should be attributed to the amazing phenomena of the Universe, when different people, at different times, tossing different coins, united by the same property of symmetry and homogeneity of the coin's manufacture, get the same result - an approximate equality of the appearance of the Coat of Arms and the Lattice. And this frequency property lies at the basis of the very existence of the Probability Theory.

Thus, nothing can be said about the outcome of each individual experiment - there is no deterministic regularity (this is the problem of probability theory), but it is possible to indicate the share of an event in a large number of experiments – the presence of static regularity.

The implementation of various experiments visually helps in consolidating the probabilistic model in the subconscious and also contributes to the involvement of students in science.

Modeling of random numbers and their application in substantiating the practical significance of probability. Let's conduct a simulation of an experiment with randomly tossing a dice. A teacher can conduct such experiments using ready-made software packages (such as Excel, Mathematica, GeoGebra) in class or by consulting with scientific researchers. One of the methods for implementing the experiment is the use of random numbers. Here you can use applications with built-in random number generators or independently generate them based on the results of scientific research (which, of course, is beyond the scope of the school curriculum, but can be used as additional material for optional classes or to involve *згшды* in scientific research). The use of ready-made software packages for generating random numbers is generally available, in connection with which we will provide an implementation of the experiment below, based on scientific research.

The Lehmer linear congruent sequence was created in 1948 from the equation of the line $y=ax+c$ in the form of a permutation of the sequence according to the recurrent formula

$$x_{n+1} = ax_n + c(\text{mod}N)(n \geq 0). \quad (1)$$

By N. Temirgaliev (Temirgaliev, 2018) is proved that if the equality $(a-1)^2=N$ is fulfilled, then the sequence (1) will be random. Let us choose $N=961$ (about 1000 tosses) and $a=32$ ($c=1$, $x_0=0$) from which we will construct the sequence (1) – $\{x_n\}_{n=1}^{961}$

0, 1, 33, 96, 190, 315, 471, 658, 876, 164, 444, 755, 136, 509, 913, 387, 853, 389, 917, 515, 144, 765, 456, 178, 892, 676, 491, ..., 484, 113, 734, 425, 147, 861, 645, 460, 306, 183, 91, 30.

This sequence includes all numbers from 0 to 960. To simulate the experiment with throwing a dice, we will write out the remainders when dividing by 6. Next, we will randomly select a part of the sequence, let it be $x_{35}, x_{36}, \dots, x_{624}$ members of the sequence modulo 6:

1, 5, 4, 4, 5, 1, 3, 1, 0, 5, 0, 2, 4, 2, 0, 0, 0, 1, 4, 1, 5, 5, 5, 0, 2, 5, 3, 2, 2, 3, 5, 2, 0, 5, 5, 0, 2, 4, 2, 1, 0, 1, 2, 5, 3, 1, 0, 1, 2, 5, 2, 0, 0, 0, 1, 3, 0, 4, 3, 3, 4, 0, 3, 1, 0, 0, 1, 2, 5, 3, 2, 1, 2, 3, 0, 4, 2, 1, 2, 3, 0, 3, 1, 0, 1, 2, 4, 1, 5, 4, 4, 5, 1, 4, 2, 1, 1, 2, 3, 0, 4, 3, 2, 3, 4, 1, 4, 3, 2, 3, 4, 1, 4, 2, 1, 2, 3, 5, 2, 0, 5, 5, 0, 2, 5, 3, 2, 2, 3, 4, 1, 5, 4, 3, 4, 5, 2, 5, 4, 3, 4, 5, 2, 5, 3, 2, 3, 4, 0, 3, 1, 0, 0, 1, 3, 0, 4, 3, 3, 4, 5, 2, 0, 5, 4, 5, 0, 3, 0, 5, 4, 5, 0, 3, 0, 4, 3, 4, 5, 1, 4, 2, 1, 1, 2, 4, 1, 5, 4, 4, 5, 0, 3, 1, 5, 5, 0, 1, 4, 1, 0, 5, 0, 1, 3, 1, 5, 4, 5, 0, 2, 5, 3, 2, 2, 3, 5, 2, 0, 5, 5, 0, 1, 4, 2, 0, 0, 1, 2, 5, 2, 1, 0, 1, 2, 4, 2, 0, 5, 0, 1, 3, 0, 4, 3, 3, 4, 0, 3, 1, 0, 0, 1, 2, 5, 3, 1, 1, 2, 3, 0, 3, 2, 1, 2, 3, 5, 3, 1, 0, 1, 2, 4, 1, 5, 4, 4, 5, 1, 4, 2, 1, 1, 1, 3, 0, 4, 2, 2, 3, 4, 1, 4, 3, 2, 3, 4, 0, 4, 2, 1, 1, 3, 5, 2, 0, 5, 5, 0, 2, 5, 3, 2, 2, 2, 4, 1, 5, 3, 3, 4, 5, 2, 5, 4, 3, 4, 5, 1, 5, 3, 2, 2, 4, 0, 3, 1, 0, 0, 1, 3, 0, 4, 3, 3, 3, 5, 2, 0, 4, 4, 5, 0, 3, 0, 5, 4, 5, 0, 2, 0, 4, 3, 3, 5, 1, 4, 2, 1, 1, 2, 4, 1, 5, 4, 4, 4, 0, 3, 1, 5, 5, 0, 1, 4, 1, 0, 5, 0, 1, 3, 1, 5, 4, 4, 0, 2, 5, 3, 2, 2, 3, 5, 2, 0, 5, 5, 1, 4, 2, 0, 0, 1, 2, 5, 2, 1, 0, 1, 2, 4, 2, 0, 5, 5, 1, 3, 0, 4, 3, 3, 4, 0, 3, 1, 0, 0, 0, 2, 5, 3, 1, 1, 1, 3, 0, 3, 2, 1, 1, 3, 5, 3, 1, 0, 0, 2, 4, 1, 5, 4, 4, 5, 1, 4, 2, 1, 0, 1, 3, 0, 4, 2, 2, 2, 4, 1, 4, 3, 2, 2, 4, 0, 4, 2, 1, 1, 2, 5, 2, 0, 5, 5, 0, 2, 5, 3, 2, 1, 2, 4, 1, 4, 3, 3, 3, 5, 2, 5, 4, 3, 3, 5, 1, 4, 3, 2, 2, 3, 0, 3, 1, 0, 0, 1, 3, 0, 4, 3, 2, 3, 5, 2, 5, 4, 4, 4, 0, 3, 0, 5, 4, 4, 0, 2, 5, 4, 3, 3, 4, 1, 4, 2, 1, 1, 2, 4, 1, 5, 4, 3, 4, 0, 3, 0, 5, 5, 5, 1, 3, 1, 5, 5, 5, 1, 3, 0, 5, 4, 4, 5, 2, 5, 3, 2, 2, 3, 5, 2

Of course, you can choose in any other combination, not necessarily in ordinal sequence, and in any quantity.

Replacing 0 with 6, we write out the frequencies of the sides 1, 2, 3, 4, 5, 6 of the dice (Table 8).

Table 8. The frequencies of the sides 1, 2, 3, 4, 5, 6 of the dice

Side of a dice	1	2	3	4	5	6
Number of drops	99	100	98	97	100	96

Next, we write out the relative frequencies - the ratio of the number of sides falling out to the total number of tosses (Table 9).

Table 9. The relative frequencies - the ratio of the number of sides falling out to the total number of tosses

1	2	3	4	5	6
$\frac{99}{590} \approx 0,1678$	$\frac{100}{590} \approx 0,1695$	$\frac{98}{590} \approx 0,1661$	$\frac{97}{590} \approx 0,1644$	$\frac{100}{590} \approx 0,1695$	$\frac{96}{590} \approx 0,1627$

From the table we see that the values of relative frequencies are close to the probabilities.

Now we will conduct a sequence of series of experiments and in each of them we will determine the frequency of occurrence of 5 (it was possible to take any number from 1 to 6). Let's take 9 series

$$\{x_n\}_{n=1}^{100}, \{x_n\}_{n=101}^{200}, \dots, \{x_n\}_{n=801}^{900},$$

consisting of 100 numbers obtained from the first 900 numbers of the sequence. Let's determine the frequencies (n_i is the number of occurrences of 5 in the i -th series)

$$\frac{n_1}{100}, \frac{n_2}{100}, \dots, \frac{n_9}{100}$$

Table 10. The frequencies n_i is the number of occurrences of 5 in the i -th series

n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9
18	15	17	17	17	12	21	17	18
$\frac{18}{100} = 0,18$	$\frac{15}{100} = 0,15$	$\frac{17}{100} = 0,17$	$\frac{17}{100} = 0,17$	$\frac{17}{100} = 0,17$	$\frac{12}{100} = 0,12$	$\frac{21}{100} = 0,21$	$\frac{17}{100} = 0,17$	$\frac{18}{100} = 0,18$

Comparing the relative frequencies of 5 with the probability $\frac{1}{6} \approx 0,1667$, of, we see that the relative frequencies are grouped around this value. These results confirm the practical significance of probability. Figure 2 shows the graphical representations of the probability and relative frequencies of 5, where the orange line shows the probability value, and the blue line shows the relative frequency values.

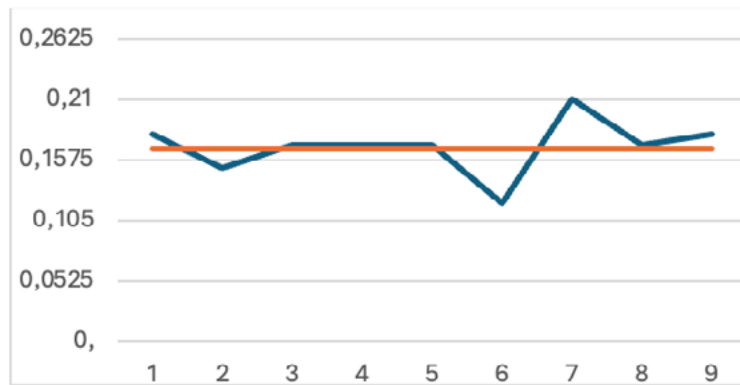


Figure 2. Probability and relative frequencies of getting 5 when throwing a dice

Now let's conduct a series of experiments with an increasing number of trials in them: the first series will include the first 50 trials of the sequence $\{x_n\}_{n=1}^{961}$, the second - the first 100 (including the previous 50), the third - the first 150, etc. up to 950. The table below shows n_i ratios of the frequency of occurrence of 5 in the i -th series to the n total number of trials in the series and the deviation of the relative frequencies from the probability value.

Table 11. ratios of the frequency of occurrence of 5 in the i -th series to the total number of trials in the series and the deviation of the relative frequencies from the probability value

n	50	100	150	200	250	300	350	400	450
n_i	9	18	21	33	44	50	57	67	78
$\frac{n_i}{n}$	0,18	0,18	0,14	0,165	0,176	0,16666	0,16285	0,1675	0,17333
$ \frac{n_i}{n} - \frac{1}{6} $	0,01333	0,01333	0,02666	0,00166	0,00933	0	0,00381	0,00083	0,00666

n	500	550	600	650	700	750	800	850	900	950
n_i	84	91	96	110	117	124	134	144	152	158
$\frac{n_i}{n}$	0,168	0,16545	0,16	0,16923	0,16714	0,16533	0,1675	0,16941	0,16888	0,16631
$ \frac{n_i}{n} - \frac{1}{6} $	0,00133	0,00121	0,00666	0,00256	0,00047	0,00133	0,00083	0,00274	0,00222	0,00035

In Fig. 3, the number of tests in the series is shown on the abscissa axis, and the corresponding relative frequencies of the 5 falling in these series are shown on the ordinate axis.

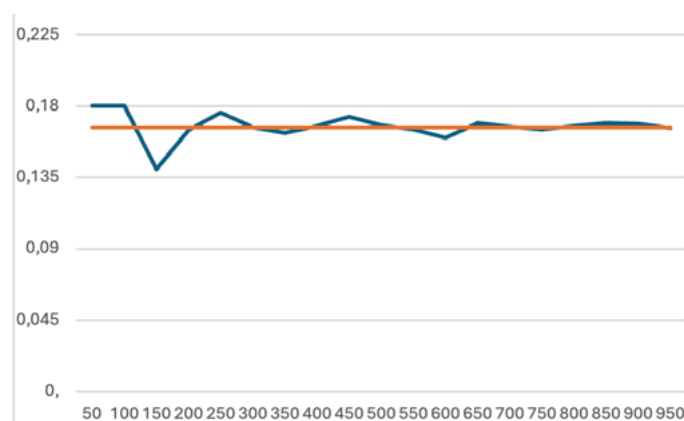


Figure 3. Probability and relative frequencies of getting 5 when throwing a dice

The results confirm the law of large numbers that relative frequencies cluster around probabilities as the number of trials increases from series to series.

Conclusion

The main task of both the teacher and the school is to bring each concept in each field of mathematics to the level of basic knowledge and to bring the student to the level of “Mathematical maturity”. To implement the same process for the elements of probability theory and mathematical statistics, it is a deep problem that does not lose its relevance, providing correct theoretical definitions and effective authors’ methods. The article presents one way to solve this problem, namely, to distinguish the main concepts in random experience, to show the general scheme of creating a probability model and an auxiliary model, as well as ways to introduce them to science.

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Conflict of interest

The authors confirm that there are no potential or actual conflicts of interest related to this study.

Contribution of the authors

The authors’ contributions are equal.

G. Taugynbayeva – Conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, writing—original draft preparation, writing—review and editing, visualization, supervision, project administration, funding acquisition.

A. Zhubanysheva – Conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, writing—original draft preparation, writing—review and editing, visualization, supervision, project administration, funding acquisition.

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Ықтималдықтар теориясы және математикалық статистика бойынша алғашқы ұғымдарды оқыту әдістемесі

Аңдатпа. Мақалада ықтималдықтар теориясы мен математикалық статистиканың алғашқы ұғымдарын оқыту мәселесі қарастырылған. Атап айтқанда, тәжірибе, тәжірибені жүзеге асыру, тәжірибе нәтижесі – элементар оқиға, оқиға, ықтималдық кеңістігі сияқты ұғымдар талқыланады. Мақалада тәжірибенің ықтималдық кеңістігін беру әдістемесі ретінде Н.Темиргалиевтің авторлық Көмекші «Маркет» моделі берілген. Стандартты «Тиын лақтыру» және ұлттық тәрбиемен байланыстырылған «Асық лақтыру» тәжірибелері мысалында көмекші және ықтималдық кеңістіктері құрастырылған. Сонымен қатар, мақалада элементар оқиғалар кеңістігі ақырлы жиын болған жағдайда ықтималдық

кеңістігін құрудың жалпы схемасын беру әдістемесі мен ықтималдықтың практикалық маңызын көрсету әдістемесі талқыланған. Сонымен қатар, мақалада кездейсоқ сандар тізбегі құрастырылып, оның негізінде симметриялы және біртекті ойын сүйегін лақтыру тәжірибесінің моделі құрылып, кездейсоқ таңдама мен эксперименттер сериясы үшін ойын сүйегінің бекітілген жағының түсу жиілігі сипатталған. Екі жағдайда да тәжірибелер саны артқан сайын жиіліктер оқиға ықтималдылығының маңайына шоғырланатындығы жайлы Үлкен сандар заңын растау негізінде білім алушы үшін алғашқы ғылыми қадамдарын жасату жолы беріледі.

Түйін сөздер: оқыту әдістемесі, ықтималдықтар теориясы және математикалық статистика, ықтималдық моделі, көмекші «Маркет» моделі, ықтималдықтың практикалық мағынасы.

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Методика преподавания начальных понятий теории вероятностей и математической статистики

Абстракт. В статье рассматривается вопрос преподавания начальных понятий теории вероятностей и математической статистики. Именно обсуждаются такие понятия, как эксперимент и его реализация, исход эксперимента – элементарное событие, событие, вероятностное пространство. Приводится предложенная Н.Темиргалиевым авторская вспомогательная модель «Маркет» для изучения вероятностного пространства эксперимента. Вспомогательные и вероятностные пространства построены на примере стандартного опыта «Подбрасывание монеты» и национальной игры «Бросание асыка». Вместе с тем в статье приводится общая схема построения вероятностного пространства в случае конечного множества элементарных событий. Далее обсуждается практическое значение вероятности. Строится последовательность случайных чисел и на ее основании моделируется эксперимент с подбрасыванием симметричной и однородной игральной кости. Находятся частота выпадения стороны игральной кости для случайной выборки и для серий экспериментов. В обоих случаях подтверждается закон больших чисел о скоплении частот с ростом количества экспериментов вокруг вероятности события. Кроме того, в статье рассматривается методика представления общей схемы построения вероятностного пространства в случае, когда пространство элементарных событий является конечным множеством, а также методика демонстрации практической значимости вероятности. Также в статье составлена последовательность случайных чисел, на основе которой построена модель эксперимента по бросанию симметричной и однородной игральной кости и описана частота выпадения фиксированной грани кубика для случайной выборки и серии экспериментов. В обоих случаях обучающемуся предоставляется возможность сделать первые научные шаги, основываясь на подтверждении закона больших чисел о стремлении частоты события к его вероятности с увеличением количества экспериментов.

Ключевые слова: методика преподавания, теория вероятностей и математическая статистика, вероятностная модель, вспомогательная модель «Маркет», практическое значение вероятности.

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